Available online at www.sciencedirect.com

ScienceDirect

International Journal of Heat and Mass Transfer 50 (2007) 960–966

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Heat transfer in a viscoelastic boundary layer flow over a stretching sheet with viscous dissipation and non-uniform heat source

M. Subhas Abel [a,*], P.G. Siddheshwar [b], Mahantesh M. Nandeppanavar [a]

[a] Department of Mathematics, Gulbarga University, Gulbarga 585 106, India

[b] Department of Mathematics, Bangalore University, Central College Campus, Bangalore 1, India

Received 16 March 2006

Available online 18 December 2006

Abstract

In this paper, visco-elastic boundary layer flow and heat transfer over a stretching sheet in presence of viscous dissipation and non-uniform heat source have been discussed. Analytical solutions of highly non-linear momentum equation and confluent hypergeometric similarity solution of heat transfer equations are obtained. Here two types of different heating processes are considered namely (i) prescribed surface temperature (PST) and (ii) prescribed wall heat flux (PHF). The effect of various parameters like visco-elastic parameter, Eckert number, Prandtl number, and non-uniform heat source/sink parameter on temperature distribution are analyzed and effect of all these parameters on wall temperature gradient and wall temperature are tabulated and discussed.

Keywords: Visco-elastic liquid; Stretching sheet; Similarity solution; Non-uniform heat source/sink; Viscous dissipation; Eckert number

1. Introduction

Sakiadis [1,2] has initiated the study of boundary layer problem assuming velocity of a boundary sheet as constant. A great deal of literature is available on the two-dimensional visco-elastic boundary layer flow over a stretching surface where the velocity of a stretching surface is assumed linearly proportional to the distance from a fixed origin. Because of numerous application of visco-elastic fluids in several industrial manufacturing processes have led renewed interest among researchers to investigate visco-elastic boundary layer flow over a stretching plastic sheet. (Rajagopal et al. [3,4], Dandapat and Gupta [5], Rollins and Vajravelu [10], Andersson [6], Lawrence and Rao [7], Char [8], Rajagopal and Gupta [9], Rao [11], Bhattacharya et al. [13] and Vajravelu and Rollins [14].) Some of the typical applications of such study are polymer sheet extrusion from a dye, glass fiber and paper production, drawing of plastic films etc.

In reality most liquids are non-Newtonian in nature, which are abundantly used in many industrial applications, such as in the manufacture of plastic films and artificial fibers, aerodynamic extrusion of plastic sheets, cooling of metallic sheets in a cooling bath, crystal growing, liquid film condensation process, continuous polymer sheet extrusion, heat treated materials traveling between a feed roll, wind up roll or on a conveyer belt, geothermal reservoirs and petroleum industries.

In view of this, the study of visco-elastic boundary layer flow problem has been further channelised to non-Newtonian fluid flow. Review of literature reveals that Rajagopal et al. [3] have considered the study of visco-elastic second order fluid flow over a stretching sheet by solving boundary layer equations numerically, this work does not take into account of the heat transfer phenomenon. Siddappa and Abel [25] have considered similar flow analysis with out

* Corresponding author. Tel.: +91 8472 245633.
E-mail address: msabel2001@yahoo.co.uk (M.S. Abel).

0017-9310/$ - see front matter © 2006 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijheatmasstransfer.2006.08.010

heat transfer in the flow of non-Newtonian fluid of Walters' liquid. Bujurke et al. [26] have presented work to analyze momentum and heat transfer phenomena in visco-elastic second order fluid over a stretching sheet with internal heat generation and viscous dissipation. An exact analytical solution of MHD flow of a visco-elastic liquid of Walters' liquid B past a stretching sheet has been presented by Andersson [6]. Numerous works are also available in the literature of viscoelastic boundary layer flow and heat transfer phenomena ([16–18,20–24,27,31,32]) but only with temperature dependent heat source/sink is considered in their analysis. Vajravelu and Nayfeh [15] studied heat transfer on a vertical sheet in a heat generating (absorbing) fluid.

Postelnicu et al. [28] examined the effect of variable viscosity on forced convection flow past a horizontal flat plate in a porous medium with internal heat generation, but in heat generation part they considered only space dependent heat source. Again Postelnicu et al. [29] studied the free convection boundary layer over a vertical permeable plate in a porous medium with internal heat generation, in this authors considered only space dependent heat generation. Postelnicu et al. [30] worked on the similarity solutions of free convection boundary layers over vertical and horizontal surfaces in porous media with internal heat generation, again here also the authors considered only space dependent heat generation. Postelnicu et al. [28–30] in these all works considered only viscous flow and space dependent internal heat generation. But Abo-Eldahab and El Aziz [12] considered the study on Blowing/Suction effect on hydromagnetic heat transfer by mixed convection from an inclined continuously stretching surface with internal heat generation/absorption, but the work considered both the space and temperature dependent heat source/sink, in a viscous flow.

In above referred literature the effect of non-uniform heat generation on heat transfer phenomenon in visco-elastic boundary liquid flow is excluded from the analysis. In non-Newtonian fluid flows like Walters' liquid B, the effect of frictional heating plays an important role in heat transfer processes. Hence in this paper, we propose to investigate the non-Newtonian visco-elastic boundary layer flow of Walters' liquid B past a stretching sheet, taking account of non-uniform heat source (Abo-Eldahab and El Aziz [12]) and frictional heating. Heat transfer characteristics are examined for two different kinds of boundary conditions, namely (i) when wall is maintained with prescribed power law temperature and (ii) when the wall is maintained with power law heat flux. Analytical solution for the flow and heat transfer are obtained in the form of confluent hyper geometric function (Kummer's function).

2. Mathematical formulation and solution

Consider the steady two-dimensional laminar flow of an incompressible visco-elastic fluid (obeying Walter's model) in the presence of a semi-infinite, impermeable flat sheet coinciding with the plane $y=0$, the flow being confined to $y>0$. Two equal and opposite forces are applied along x-axis, so that the surface is stretched, keeping the origin fixed. Under the usual boundary layer assumptions, the basic boundary layer equations governing the flow of Walters' Liquid B can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-k_0\left\{u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right\} \tag{2}$$

where x and y represent horizontal and transverse directions respectively, and u, v, respectively, are the velocities along x and y directions. υ is the kinematic viscosity, k_0 is the co-efficient of viscoelasticity. In deriving these equations, it is assumed, in addition to the usual boundary layer approximations, that the contribution due to the normal stress is of the same order of magnitude as the shear stress. Hence both υ and k_0 are of the order of the square of the boundary layer thickness.

The boundary conditions are

$$\begin{aligned}&u_w(x)=bx,\quad v=0,\quad \text{at } y=0,\\&u\to 0,\quad \text{as } y\to\infty.\end{aligned} \tag{3}$$

with $b>0$, this is known as stretching rate. Eqs. (1) and (2), subjected to boundary condition (3), admit self-similar solution in terms of the similarity function f and the similarity variable η defined by

$$u=bxf_\eta(\eta),\quad v=-\sqrt{b\upsilon}f(\eta),\quad \eta=\sqrt{\frac{b}{\upsilon}}y, \tag{4}$$

where suffix denotes the derivative w.r.t η. Clearly u and v as defined above satisfy the continuity equation (1) and (2) becomes

$$f_\eta^2-ff_{\eta\eta}=f_{\eta\eta\eta}-k_1\left\{2f_\eta f_{\eta\eta\eta}-ff_{\eta\eta\eta\eta}-f_{\eta\eta}^2\right\}, \tag{5}$$

where $k_1=\frac{k_0 b}{\upsilon}$, is the visco-elastic parameter.

Similarly boundary conditions (Eq. (3)) become,

$$\begin{aligned}&f_\eta(\eta)=1,\quad f(\eta)=0\quad \text{at } \eta=0,\\&f_\eta(\eta)\to 0,\quad \text{as } \eta\to\infty,\end{aligned} \tag{6}$$

It is to be noted that the boundary condition Eq. (6) is not sufficient to solve Eq. (5) uniquely. So making use of boundary condition Eq. (6), Rajagopal et al. [4] obtained the corresponding solution of Eq. (5), which is an exact solution of Eq. (5), satisfying the boundary conditions (6) and is given by

$$f(\eta)=\frac{1-e^{-\alpha\eta}}{\alpha},\quad \text{with } \alpha=\sqrt{\frac{1}{1-k_1}}. \tag{7}$$

Obviously, $0<k_1<1$.

Therefore, the velocity components are

$$u=bxe^{-\alpha\eta},\quad \text{and}\quad v=-\sqrt{b\upsilon}\left(\frac{1-e^{-\alpha\eta}}{\alpha}\right). \tag{8}$$

In the Section 3 we consider the heat transfer in the considered flow.

3. Heat transfer

Since the fluid we considered in the analysis is visco-elastic, the energy will be stored in the fluid by means of frictional heating due to viscous dissipation. So we take account of this. However, we assume that fluid possesses strong viscous property than elastic property. Also the effect of elastic deformation terms might not be sufficient as the momentum boundary layer equation is valid at low shear rate and small values of elastic parameter [3,5,18]. So in view of this we may neglect the contribution of heat energy due to elastic deformation. Hence the governing boundary layer heat transport equation in the presence of viscous dissipation and non-uniform internal heat generation/ absorption for two-dimensional flow is

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2+q''', \tag{9}$$

where k is the thermal conductivity, ρ is the density, T is the temperature, c_p is the specific heat at constant pressure μ is the viscosity and q''' is the space- and temperature-dependent internal heat generation/absorption (non-uniform heat source/sink) [12] which can be expressed in simplest form as

$$q'''=\left(\frac{ku_w(x)}{x\upsilon}\right)[A*(T_w-T_\infty)f'(\eta)+B^*(T-T_\infty)], \tag{10}$$

where A^* and B^* are parameters of space- and temperature-dependent internal heat generation/absorption. It is to be noted that $A^*>0$ and $B^*>0$ correspond to internal heat generation while $A^*<0$ and $B^*<0$ correspond to internal heat absorption. The solution of Eq. (9) depends on the nature of the prescribed boundary conditions. Two types of heating processes are considered as discussed below.

3.1. Case A: Prescribed power law surface temperature (PST case)

For this heating process, the prescribed surface temperature is assumed to be a quadratic function of x and is given by

$$\begin{aligned}&T=T_w=T_\infty+A\left(\frac{x}{l}\right)^2\quad \text{at } y=0,\\&T\to T_\infty\quad \text{as } y\to\infty,\end{aligned} \tag{11}$$

where T_w is the temperature of the wall and T_∞ is the temperature out side the dynamic region. The constant A depends on the thermal properties of the liquid and $l=\sqrt{\frac{\upsilon}{b}}$ is a characteristic length. We now define a dimensionless scaled temperature as

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \tag{12}$$

where $T-T_\infty=A\left(\frac{x}{l}\right)^2\theta(\eta)$.

Eq. (9) on using Eqs. (11) and (12) can be transformed to the following equation

$$\begin{aligned}&\theta_{\eta\eta}(\eta)+Prf(\eta)\theta_\eta(\eta)-(2Prf_\eta(\eta)-B^*)\theta(\eta)\\&\quad=-(EPrf_{\eta\eta}^2+A^*f_\eta),\end{aligned} \tag{13}$$

where $E=\frac{b^2l^2}{AC_p}$ (Eckert number), $Pr=\frac{\mu C_p}{k}$ (Prandtl number).

Using Eq. (12) in Eq. (13) the boundary conditions read as

$$\begin{aligned}&\theta(\eta)=1\quad \text{at } \eta=0,\\&\theta(\eta)\to 0\quad \text{as } \eta\to\infty.\end{aligned} \tag{14}$$

The solution of Eq. (13), subject to boundary conditions (14), can be obtained in terms of hypergeometric Kummer's function [19] as

$$\begin{aligned}\theta(\eta)=&c_1(\mathrm{e}^{-\alpha\eta})^{\frac{a_0+b_0}{2}}M\left(\frac{a_0+b_0}{2}-2,1+b_0,-\frac{Pr}{\alpha^2}\mathrm{e}^{-\alpha\eta}\right)\\&+c_2\mathrm{e}^{-\alpha\eta}+c_3\mathrm{e}^{-2\alpha\eta},\end{aligned} \tag{15}$$

where

$$a_0=\frac{Pr}{\alpha^2},\quad b_0=\sqrt{a_0^2-\frac{4B^*}{\alpha^2}},\quad c_1=\frac{1-(c_2+c_3)}{M(\frac{a_0+b_0}{2}-2,1+b_0,-\frac{Pr}{\alpha^2})},$$
$$c_2=\frac{-A^*}{(4\alpha^2-2Pr+B^*)}\quad \text{and}\quad c_3=\frac{-E\alpha^2 Pr}{(4\alpha^2-2Pr+B^*)}.$$

The non-dimensional wall temperature gradient derived from Eq. (15) reads as

$$\begin{aligned}\theta'(0)=&c_1\left[-\alpha\left(\frac{a_0+b_0}{2}\right)M\left(\frac{a_0+b_0-4}{2},1+b_0;-\frac{Pr}{\alpha^2}\right)\right.\\&\left.+\left(\frac{a_0+b_0-4}{2(1+b_0)}\right)\frac{Pr}{\alpha}M\left(\frac{a_0+b_0-2}{2},2+b_0;-\frac{Pr}{\alpha^2}\right)\right]\\&-c_2\alpha-c_3\alpha.\end{aligned} \tag{16}$$

3.2. Case B: Prescribed power law surface heat flux (PHF case)

The power law heat flux on the wall surface is considered to be a quadratic power of x in the form

$$\begin{aligned}&-k\frac{\partial T}{\partial y}=q_w=D\left(\frac{x}{l}\right)^2\quad \text{at } y=0,\\&T\to T_\infty\quad \text{as } y\to\infty,\end{aligned} \tag{17}$$

where D is a constant, k is the thermal conductivity and l is as defined earlier. We now define a dimensionless, scaled temperature $g(\eta)$ as

$$g(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \tag{18}$$

where

$$T_w-T_\infty=\frac{D}{k}\left(\frac{x}{l}\right)^2\sqrt{\frac{\upsilon}{b}}. \tag{19}$$

Using Eqs. (17) and (18), (9) can be written in terms of g as

$$g_{\eta\eta}(\eta) + Prf(\eta)g_\eta(\eta) - (2Prf_\eta(\eta) - B^*)g(\eta) = -(EPrf_{\eta\eta}^2 + A^*f_\eta), \quad (20)$$

where $E = \frac{kb^2 l^2 \sqrt{\frac{b}{\upsilon}}}{DC_p}$ (Eckert number) and the boundary conditions take the form

$$g_\eta(\eta) = -1 \quad \text{at } \eta = 0, \qquad g(\eta) \to 0 \quad \text{as } \eta \to \infty, \quad (21)$$

with $f(\eta)$ as defined earlier in the PST case. The solution of Eq. (20), subject to the boundary condition (21), can be obtained in terms of hypergeometric Kummer's function [19] in the form

$$g(\eta) = c_4 e^{-\alpha\left(\frac{a_0+b_0}{2}\right)\eta} M\left(\frac{a_0+b_0}{2} - 2, 1 + b_0; -\frac{Pr}{\alpha^2} e^{-\alpha\eta}\right) + c_2 e^{-\alpha\eta} + c_3 e^{-2\alpha\eta}, \quad (22)$$

where a_0, b_0, c_2 and c_3 are as defined earlier in the PST case and c_4 is given by

$$c_4 = \frac{(c_2 + 2c_3)\alpha - 1}{\left[-\alpha\left(\frac{a_0+b_0}{2}\right) M\left(\frac{a_0+b_0-4}{2}, 1 + b_0; -\frac{Pr}{\alpha^2}\right) + \left(\frac{a_0+b_0-4}{2(1+b_0)}\right)\left(\frac{Pr}{\alpha}\right) M\left(\frac{a_0+b_0-2}{2}, 2 + b_0; -\frac{Pr}{\alpha^2}\right)\right]}. \quad (23)$$

The non-dimensional wall temperature derived from Eq. (22) reads as

$$g(0) = c_4 M\left(\frac{a_0+b_0}{2} - 2, 1 + b_0; -\frac{Pr}{\alpha^2}\right) + c_2 + c_3. \quad (24)$$

Now we proceed to the discussion of results of the undertaken study.

4. Results and discussion

A boundary layer problem for momentum and heat transfer with space and temperature dependent heat source in viscoelastic fluid flow over a stretching sheet is examined in this paper. The boundary layer equations of momentum and heat transfer are solved analytically and the different analytical expressions are obtained for non-dimensional temperature profiles for two general cases of boundary conditions namely (i) PST Case (ii) PHF Case. Explicit analytical expressions are also obtained for dimensionless temperature gradient $\theta'(0)$ and g(0). Numerical computations of results are demonstrated in Figs. 1–5 for PST and PHF cases respectively. The parameters that arise in the study are viscoelastic parameter k_1, Prandtl number Pr, Eckert number E, space-dependent heat source/sink parameter A^* and temperature-dependent heat source/sink parameter B^*. The parameters k_1, E, and Pr are well known. The parameters A^* and B^* are not so large quantities. We now proceed with the discussion of results.

Fig. 1(a) is drawn for temperature profile $\theta(\eta)$ versus η from the sheet, for the PST case, for different values of k_1 and Fig. 1(b) is the graphical representation of the temperature profile $g(\eta)$ versus η for the PHF case, for different values of k_1. From these figures it is apparent that the temperature is unchanged at the wall with the change of physical parameters in PST case and we also observe that the temperature increases with increase in the value of k_1, in both PST and PHF cases.

This is due to the fact that an increase of visco-elastic normal stress gives rise to thickening of thermal boundary layer.

Figs. 2(a) and 2(b) depict the temperature profiles $\theta(\eta)$ and $g(\eta)$ versus η from the sheet, for different values of Pr. We infer from these figures that temperature decreases with increase in Pr which implies viscous boundary layer is thicker than the thermal boundary layer. Temperature in both PST and PHF cases asymptotically approaches to zero in free stream region.

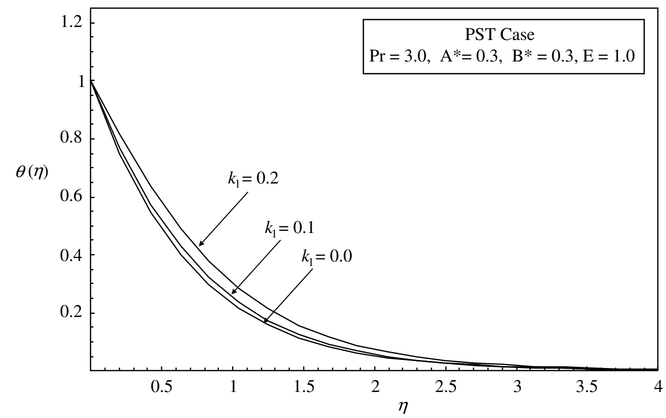

Fig. 1(a). Effect of visco-elasticity (k_1) on temperature distribution in PST case.

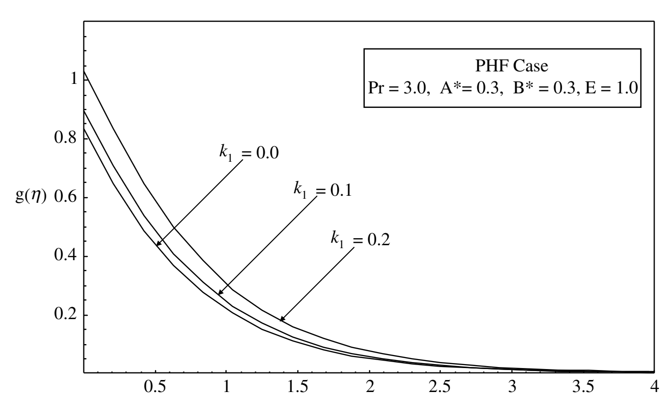

Fig. 1(b). Effect of visco-elasticity (k_1) on temperature distribution in PHF case.

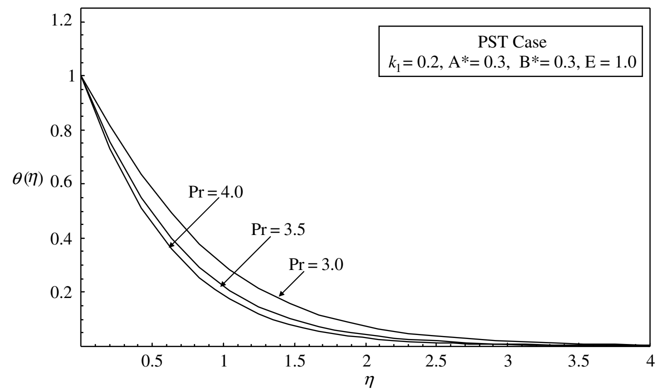

Fig. 2(a). Effect of Prandtl number (Pr) on temperature distribution in PST case.

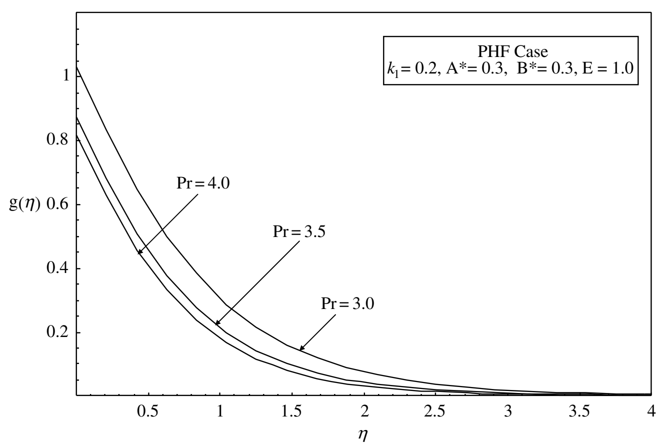

Fig. 2(b). Effect of Prandtl number (Pr) on temperature distribution in PHF case.

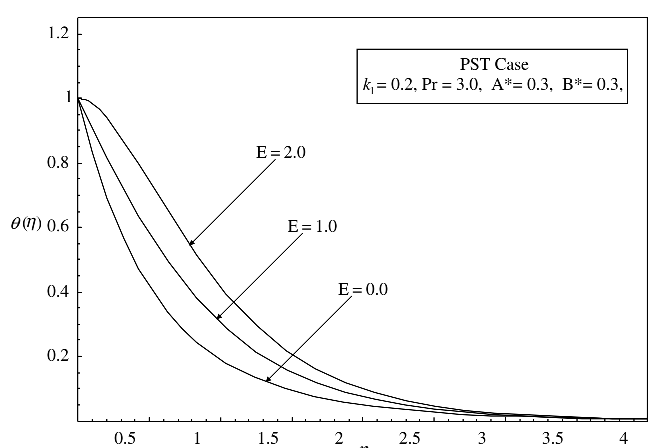

Fig. 3(a). Effect of Eckert number (E) on temperature distribution in PST case.

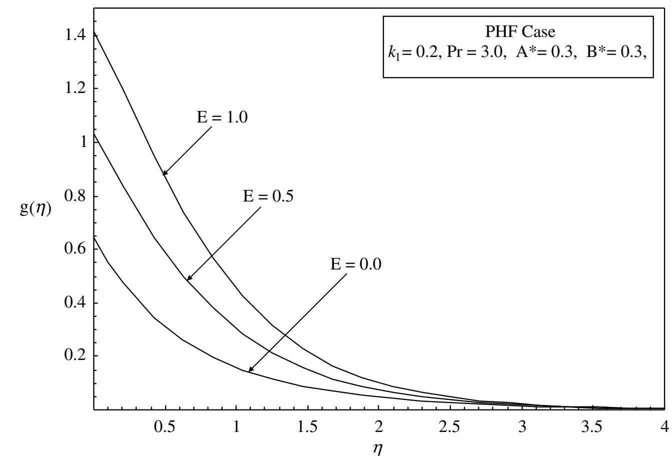

Fig. 3(b). Effect of Eckert number (E) on temperature distribution in PHF case.

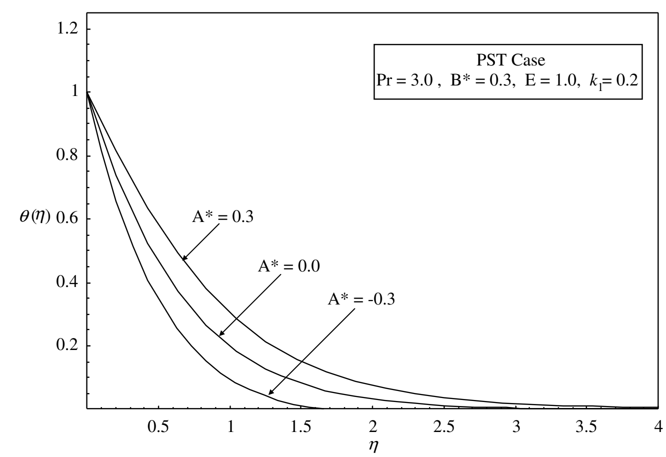

Fig. 4(a). Effect of non-uniform heat source/sink parameter (A^*) on temperature distribution in PST case.

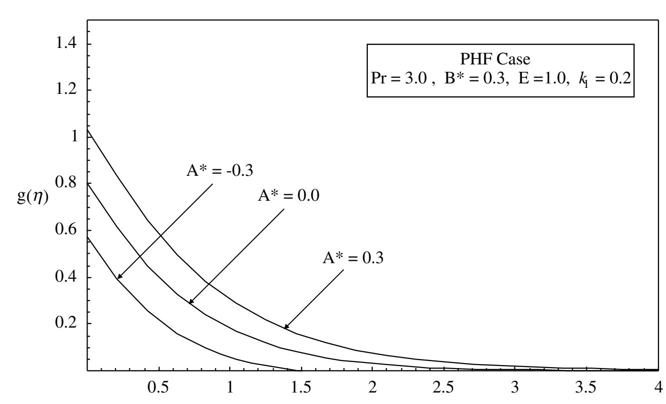

Fig. 4(b). Effect of non-uniform heat source/sink parameter (A^*) on temperature distribution in PHF case.

The Figs. 3(a) and 3(b) show the temperature distribution $\theta(\eta)$ and $g(\eta)$ versus η from the sheet, for different values of Eckert number (E) for both PST and PHF cases, respectively. By analyzing the graphs it reveals that the effect of increasing values of E is to increase the temperature distribution in flow region in both PST and PHF cases. This is due to the fact that heat energy is stored in the

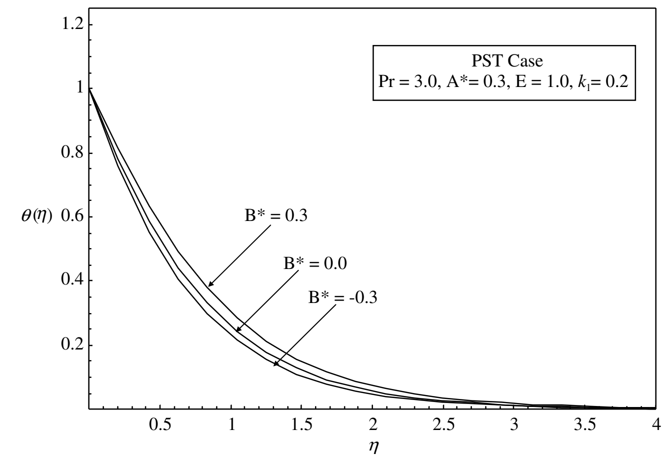

Fig. 5(a). Effect of non-uniform heat source/sink parameter (B^*) on temperature distribution in PST case.

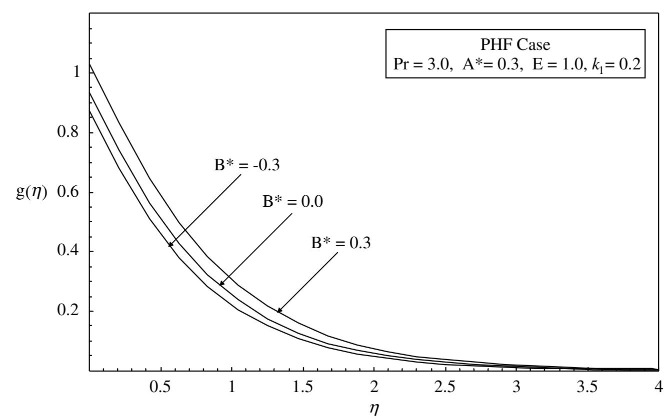

Fig. 5(b). Effect of non-uniform heat source/sink parameter (B^*) on temperature distribution in PHF case.

liquid due to the frictional heating. The effect of increasing E, is to enhance the temperature at any point and this is true in both cases.

Figs. 4(a) and 4(b), are graphs of temperature profiles $\theta(\eta)$ and $g(\eta)$ versus distance η, for different values of A^*. For $A^* > 0$, it can be seen that the thermal boundary layer generates the energy, and this causes the temperature $\theta(\eta)$ and $g(\eta)$ of the fluid to increase with increase in the value of $A^* > 0$ (heat source), where as for $A^* < 0$ (absorption) the temperature $\theta(\eta)$ decreases with increase in the value of A^*.

Figs. 5(a) and 5(b) depict the temperature profiles $\theta(\eta)$ and $g(\eta)$ versus distance η, for different values of B^*. The explanation on the effect of B^* is similar to that given for A^*.

The heat transfer phenomena is also analyzed from the numerical results of two physical parameters, namely (i) wall temperature gradient $\theta'(0)$ in PST Case and (ii) wall temperature $g(0)$ in PHF case and the same are presented in Table 1. Analyzing this table reveals that the effect of increasing the value of k_1 is to increase the wall temperature gradient $\theta'(0)$ in PST Case and wall temperature $g(0)$ in PHF Case. The effect of Pr is to decrease $\theta'(0)$ and $g(0)$ significantly. The effect of E (Eckert number) is to decrease $\theta'(0)$ and $g(0)$ and increasing the value of A^* and B^*(negative to positive) is clearly to increase both $\theta'(0)$ and $g(0)$. On comparison of temperature distribution of PST and PHF, it is apparent that the PST boundary condition succeeds in keeping viscoelastic cooling liquid warmer than in case when PHF boundary condition is applied. It may be therefore be inferred that the PHF boundary condition is better suited for faster cooling of stretching sheet. The results of PST and PHF cases infer that the boundary layer temperature is quantitatively higher in PST case as compared to PHF case and the

Table 1
Value of wall temperature gradient $\theta'(0)$ (for PST Case) and wall temperature $g(0)$ (for PHF Case), for different values of E, k_1, Pr, A^* and B^*

E	k_1	Pr	A^*	B^*	PST Case-$\theta'(0)$	PHF Case-$g(0)$
0.0	0.2	4.0	0.3	0.3	2.65822	0.41832
0.02					2.59404	0.426306
0.5					1.05386	0.617967
0.02	0.0	4.0	0.3	0.3	2.68986	0.406411
	0.1				2.65169	0.414147
	0.2				2.59404	0.426306
0.02	0.2	3.0	0.3	0.3	1.71759	0.652451
		3.5			2.29203	0.488533
		4.0			2.59404	0.426306
0.02	0.2	4.0	−0.3	0.3	2.97909	0.291238
			0.0		2.78657	0.358772
			0.3		2.59404	0.426306
0.02	0.2	4.0	0.3	−0.3	2.73959	0.401513
				0.0	2.66907	0.4131630
				0.3	2.59404	0.426306

results are in tune with what happens in regions away from the sheet.

Acknowledgement

The authors are thankful to DST-New Delhi, for financial support to pursue this work under Major Research Project (Grant No. SR/S4/MS: 198/03).

References

[1] B.C. Sakiadis, Boundary layer behavior on continuous solid surfaces: I Boundary layer equations for two dimensional and axisymmetric flow, AIChE. J. 7 (1961) 26–28.

[2] B.C. Sakiadis, Boundary layer behavior on continuous solid surfaces: II Boundary layer on a continuous flat surface, AIChE. J. 7 (2) (1961) 221–225.

[3] K.R. Rajagopal, T.Y. Na, A.S. Gupta, Flow of a visco-elastic fluid over a stretching sheet, Rheol. Acta. 23 (1984) 213–215.

[4] K.R. Rajagopal, T.Y. Na, A.S. Gupta, A non similar boundary layer on a stretching sheet in a non-Newtonian fluid with uniform free stream, J. Math. Phys. Sci. 21 (2) (1987) 189–200.

[5] B.S. Dandapat, A.S. Gupta, Flow and heat transfer in a viscoelastic fluid over a stretching sheet, Int. J. Non-linear Mech. 24 (3) (1989) 215–219.

[6] H.I. Andersson, MHD flow of a viscoelastic fluid past a stretching surface, Acta. Mech. 95 (1992) 227–230.

[7] P.S. Lawrence, B.N. Rao, Heat transfer in the flow of viscoelastic fluid over a stretching sheet, Acta. Mech. 93 (1992) 53–61.

[8] M.I. Char, Heat and mass transfer in a hydromagnetic flow of viscoelastic fluid over a stretching sheet, J. Math. Anal. Appl. 186 (1994) 674–689.

[9] K.R. Rajagopal, A.S. Gupta, Exact solution for the flow of a non-Newtonian fluid past an infinite porous plate, Meccanica 19 (1984) 158.

[10] D. Rollins, K. Vajravelu, Heat transfer in a second order fluid over a continuous stretching surface, Acta Mech. 89 (1991) 167–178.

[11] B.N. Rao, Technical note: Flow of a fluid of second grade over a stretching sheet, Int. J. Non-Linear Mech. 31 (4) (1996) 547–550.

[12] Emad M. Abo-Eldahab, Mohamed A. El Aziz, Blowing/suction effect on hydromagnetic heat transfer by mixed convection from an inclined continuously stretching surface with internal heat generation/absorption, Int. J. Therm. Sci. 43 (2004) 709–719.

[13] S. Bhattacharya, A. Pal, A.S. Gupta, Heat transfer in the flow of a visco-elastic fluid over a stretching surface, Heat Mass Transfer 34 (1998) 41–45.

[14] K. Vajravelu, D. Rollins, Heat transfer in electrically conducting fluid over a stretching surface, Int. J. Non-Linear Mech. 27 (2) (1992) 265–277.

[15] K. Vajravelu, J. Nayfeh, Convective heat transfer at a stretching sheet, Acta Mech. 96 (1–4) (1993) 47–54.

[16] H.R. Nataraja, M.S. Sharma, B. Nageswara Rao, Non-similar solutions for flow and heat transfer in a visco-elastic fluid over a stretching sheet, Int. J. Non-Linear Mech. 33 (1998) 357–361.

[17] R.M. Sonth, S.K. Khan, M.S. Abel, K.V. Prasad, Heat and mass transfer in a visco- elastic fluid flow over an accelerating surface with heat source/sink and viscous dissipation, Heat Mass Transfer 38 (2002) 213–220.

[18] P.G. Siddheshwar, U.S. Mahabaleshwar, Effects of radiation and heat source on MHD flow of a visco-elastic liquid and heat transfer over a stretching sheet, Int. J. Non-linear Mech. 40 (2005) 807–821.

[19] M. Abramowitz, L.A. Stegun, Handbook of Mathematical Functions, National Bureau of standards/Amer. Math. Soc. 55, Providence, RI, 1972.

[20] H.I. Andersson, R.O. Hansen, B. Holmedal, Diffusion of a chemically reactive species from a stretching sheet, Int. J. Heat Transfer 37 (4) (1994) 659–664.

[21] W.D. Chang, The non-uniqueness of the flow of a viscoelastic fluid over a stretching sheet, Quart. Appl. Math. 47 (2) (1989) 365–366.

[22] K.V. Prasad, S. Abel, S.K. Khan, Momentum and heat transfer in viscoelastic fluid flow in a porous medium over a non-isothermal stretching sheet, Inter. J. Numer. Method Heat Fluid Flow 10 (8) (2000) 786–801.

[23] K.V. Prasad, S. Abel, S.K. Khan, P.S. Datti, Non-Dracy forced convective heat transfer in a viscoelastic fluid flow over a non-isothermal stretching sheet, J. Porous Media 5 (1) (2002) 41–47.

[24] S.K. Khan, M.S. Abel, R.M. Sonth, Viscoelastic MHD flow heat and mass transfer in a over porous stretching sheet dissipation of energy and stress work, Heat Mass Transfer 40 (2003) 47–57.

[25] B. Siddappa, S. Abel, Non-Newtonian flow past a stretching plate, ZAMP 36 (1985) 890–892.

[26] N.M. Bujurke, S.N. Biradar, P.S. Hiremath, Second order fluid flow past a stretching sheet with heat transfer, ZAMP 38 (1987) 890–892.

[27] P. Sam Lawrence, B.N. Rao, Reinvestigation of the non-uniqueness of the flow of visco-elastic fluid flow over a stretching sheet, Quat. Appl. Math. 51 (3) (1993) 401–404.

[28] A. Postelnicu, T. Grosan, I. Pop, The effect of variable viscosity on forced convection flow past a horizontal flat plate in a porous medium with internal heat generation, Mech. Res. Commun. 28 (2001) 331–337.

[29] A. Postelnicu, T. Grosan, I. Pop, Free convection boundary layer over a vertical permeable plate in a porous medium with internal heat generation, Int. Commun. Heat Mass Transfer 27 (5) (2000) 729–738.

[30] A. Postelnicu, I. Pop, Similarity solutions of free convection boundary layers over vertical and horizontal surfaces in porous media with internal heat generation, Int. Commun. Heat Mass Transfer 26 (8) (1999) 1183–1191.

[31] P.S. Datti, K.V. Prasad, M.S. Abel, A. Joshi, MHD visco-elastic fluid flow over a non- isothermal stretching sheet, Int. J. Eng. Sci. 42 (2004) 935–946a.

[32] S. Abel, P.H. Veena, K. Rajagopal, V.K. Paveen, Non-Newtonian magnetohydrodynamic flow over a stretching sheet with heat and mass transfer, Int. J. Non Linear Mech. 39 (2004) 1067–1078.